Lecture 10/20/23: Compositions of Functions

↑ "function cannabalism"

Composition of functions: the composition of two functions f and g is

"f composed with g" $f \circ g = f(g(x))$ ← f eats g.

"g composed with f" $g \circ f = g(f(x))$ ← g eats f

order matters

Warning: In general

$$f \circ g \neq g \circ f.$$

Ex: Let $f(x) = 3x - 1$ and $g(x) = \sqrt{x+3}$ find $g(f(x))$.

$$g(f(x)) = g(3x-1) = \sqrt{(3x-1)+3} = \sqrt{3x+2}$$

Let's find $f(g(x))$ for fun:

$$f(g(x)) = f(\sqrt{3x-1}) = 3(\sqrt{3x-1}) - 1$$

$$\neq g(f(x))!$$

Ex Let $f(x) = x+1$ $g(x) = \frac{x^2-1}{x(x+1)}$. Find $g(f(x))$.

$$g(f(x)) = g(x+1) = \frac{(x+1)^2 - 1}{(x+1)((x+1)-1)} = \frac{(x+1)^2-1}{(x+1)x}$$

Ex:

x	0	1	2	3	4	5
$p(x)$	5	0	13	6	2	1
$q(x)$	3	1	4	9	2	0
$p\circ q(x)$	6	0	2	?	?	5
					13	

$p\circ q(0) = p(q(0)) = p(3) = 6$

$p\circ q(1) = p(q(1)) = p(1) = 0$

$p\circ q(2) = p(q(2)) = p(4) = 2$

$(p\circ q)(3) = p(q(3)) = p(9)$ ← we don't know what this is !!

$p\circ q(4) = p(2) = 13$

$(p\circ q)(5) = p(q(5)) = p(0) = 5$

Ex: #7a $y = u(v(x))$ $u(x)$ is given. Find a possible formula for $v(x)$. This is decomposing functions

$y = e^{x^2+1}$ $\quad u(x) = e^x$

Sol: $u(v(x)) = y = e^{x^2+1}$] what we know!

$u(x) = e^x$ so $u(v(x)) = e^{v(x)}$. Hence,

$$e^{v(x)} = y = e^{x^2+1}$$

So $v(x) = x^2+1$.

~~Ex #8a)~~ g'

Don't do #8 this is silly. There is not something like this in the HW. #8 on HW is similar to #7

Ex (HW)

a) Find a formula for $g(x)$ given $g(h(x)) = (x+2)^2$ and $h(x) = x+2$.

Sol: $g(x+2) = g(h(x)) = (x+2)^2$

So $g(x+2) = (x+2)^2$

~~way #1~~ lets set $x+2 = y$ sorta a dumby var.

Then $g(y) = y^2$.

Hence $\boxed{g(x) = x^2}$!!

b) $g(h(x)) = \sqrt{6x^2+4}$

Find $g(x)$ if $h(x) = x^2$

$g(x^2) = g(h(x)) = \sqrt{6x^2+4}$

Set $x^2 = y$

$g(y) = \sqrt{6y+4}$

So $\boxed{g(x) = \sqrt{6x+4}}$!!